An Assessment Of High School Leavers’ Development of Mathematical Thinking

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Abstract

This descriptive design study evaluated the mathematical thinking attainment level of 649 High School leavers, ages 18 to 19 years, in the context of their preparation for facing the challenges of the tertiary level math curriculum. The findings depict students’ low level of mathematical thinking attainment regarding their paucity in critical thinking and a lack of heuristics repertoire as a guide to employing their contextual knowledge in solving fundamental non-routine problems. These findings are a matter of concern as there seems to be a mutual exclusivity between the content learned in school and the ability of students to think mathematically.

Keywords: Mathematical Thinking; Non-Routine; Problem-Solving; Heuristics

1.0 Introduction

After spending approximately 11-13 years of formal math learning, high school students tend to carry along with them a vast amount of ‘learned’ content knowledge in the quest to face challenges and preparation for college/tertiary level learning. Such mathematical content knowledge is often absorbed in a tacit rather than explicit manner, and students tend to miss out on the sense of understanding. Featherstone et al., (1995) note that due to the rote learning mechanism, students can manoeuvre around the given mathematical routines or methodologies in completing the tasks assigned to them, but may still not be able to comprehend or appositely understand the cognitive processes behind it. This scenario of school mathematics deficiency, also prevalent in the Malaysian context (Aida, 2015; Radzi, Abu, & Mohamad, 2009; Parmjit & Whie, 2006), is seen on a worldwide scale, thereby suggesting an unfortunate divorce between notions of common sense and mathematical knowledge acquired in school. These findings were elucidated long ago and still prevail in today’s math classroom settings. This concern was probably best summed up by Steffe (1994) decades ago when he said:

“The current notion of school mathematics is based almost exclusively on formal mathematical procedures and concepts that are very remote from the conceptual world of the learners who are to learn them. (p. 5).”

This elucidation was about three decades ago. However, is it still prevalent at current times? We need to assess students learning in order to assess the current impact of instructions on students learning, especially in the context of High School leavers preparation in
facing the challenges of tertiary level math curriculum. Thus, this study was conducted with the aim to investigate high school leavers’ development of mathematical thinking in assessing their preparedness for tertiary level education.

2.0 Literature Review

It is vital for college-going students to actively re-assess the mathematical knowledge they bring forth from their school days to understand the workings behind college mathematics appositely and its link to applied mathematics, for instance, in STEM education. Researchers have questioned the low-level mastery of high school leavers' mathematical knowledge, and its effect on their transition to tertiary level is a matter of concern. The findings show that these students faced great difficulty conceptualizing fundamental topics of knowledge and using edge of division learned in primary schools. It is well known that the ally constructed knowledge Nasir, participants to be in the ... Nasir, et. al., (2021) among first-year first students majoring in math education found that their semester end results do not strut in tandem with their mathematical thinking development. They elucidated that these students have a superfluous understanding of mathematics and faced great difficulty in applying previously learnt math formulas in solving problems.

Since 2010, several efforts have been taken by the Education Ministry of Malaysia to bring about positive changes in the Malaysian education system. In particular, the aim of these reforms from the mathematics perspective was mainly due to the substandard ranking of Malaysian students' participation in the international studies of Trends in Mathematics and Science Studies (TIMSS) and Programme for International Student Assessment (PISA). These findings over the decade suggest the achievement of Malaysian students in both Mathematics and Science was way below the international benchmark. Thus, inadvertently, it led to the introduction of the new Standard Based Curriculum for Primary Schools (KSSR) and Standard Based Curriculum for Secondary Schools (KSSM) in 2019 under the new Malaysian Education Blueprint 2013-2025 for both primary and secondary education. The previous education minister of Malaysia, Datuk Seri Mahdzir Khalid, stated in a press statement that "The new curriculum emphasizes the teaching centered on the students and focuses more on problem-solving, project-based assignments, updating subjects or themes and implementing formative assessments" (The Star Online, December 31, 2016). The thrust of this curriculum for students was the embedment of a balanced set of knowledge and using critical thinking skills to develop creativity and solve problems. One of the goals of this blueprint is for Malaysian participants to be in the top third ranking by 2025. How much these curriculum reforms have impacted students’ mathematical thinking development, especially among high school leavers in their pursuit of tertiary level education after five years of implementation of KSSM, must be further deliberated.

According to Devlin (2013), mathematical thinking is a process of learning a mathematical idea by dismembering and breaking it down to its numerical and auxiliary roots and thought processes based on the problem posed. These problems must also be intellectually stimulating yet within their potential construction range. Zacharopoulosa, et. al., (2021) posited that memorising inhibits cognitive growth development with negative consequences on learner’s future performance. Polya (2004) suggested that to fuel growth in students’ higher-order thinking skills, they should use non-routine problems for their cognitive growth. “Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed to solve routine problems, even when the knowledge and the skills required for their solution have been learned over the years” (Mullis et al., 2003, p. 32). The following two examples exemplify the cognitive demands for a seemingly effortless solution yet challenging.

Example 1: A lily pad doubles in size (area) every minute. It takes 48 minutes for the lily pad to cover an entire pond. How long did it take for the lily pad to cover half of the pond? (Correct response = 47 minutes; Intuitive response = 24 minutes)

Frederick (2005) first used this example to investigate learners’ cognitive reflection in using analytic reasoning processes towards its solution. Findings show approximately three-fifths of respondents produce an incorrect, erroneous response of 24 minutes instead of 47 minutes.

Example 2: The following algorithm shows 0/0 = 2?

\[
0 = \frac{10^2 - 10^2}{100 - 100} = \frac{(10+10)(10-10)}{10(10-10)} = \frac{20}{10} = 2
\]

Is the above correct? Provide your reasoning. This example was taken from the research done by Parmjit et al., (2016) in examining students’ mathematical reasoning based on their prior learned knowledge of division learned in primary schools. It is well known that the solution for the above is undefined. Their findings revealed that four-fifths of the learners could not provide mathematical reasoning for the given solution of 2 as the answer. The non-routine examples above denote a mode of thinking that propels the mind’s curiosity, where there is no singular or fixed way to go about it, subsequently calling for the application of creativity and previously constructed knowledge in a novel and unfamiliar situation.
George Pólya, in his book entitled ‘How to solve it,’ elucidates, "A great discovery solves a great problem, but there is a grain of discovery in the solution to any problem." (p. v). By ‘grain of discovery,’ it is meant to point to cognitive mathematical thinking. Is this grain of discovery innate in High school learners in preparation for tertiary level? Thus, this research investigated high school leavers’ development of mathematical thinking in assessing their preparedness for tertiary-level education. The objectives of study are as follow:

1. To examine the extent of students’ attainment in the Mathematical Thinking Test.
2. To determine if there a significant impact of Additional Math Grades on students’ Mathematical Thinking Test (MTT) scores.
3. To examine the difficulties students face in cognizing the non-routine task in the Mathematical Thinking Test.

3.0 Methodology

A purely quantitative approach was utilized in this study, using a descriptive design among 649 randomly selected secondary school graduates aged 18 to 19 who registered for a Diploma in engineering program in a public university in Malaysia. This design was utilised, as elucidated by Kothari (2004), to “describes, records and interpret phenomena without manipulation of variables that either exists or previously existed” (p.120). On the other hand, the sample size was determined according to the sampling formula: $n = \left[ \frac{z^2 \times p \times (1 - p)}{e^2} \right] + 1$ where: $z = 2.576$ for a confidence level ($\alpha$) of 99%, $p = $ proportion (expressed as a decimal), $N = $ population size, $e = $ margin of error. This yielded a sample size of 623. An instrument developed by Parmjit et al. (2016) was adapted to measure mathematical thinking attainment among these high school leavers. This paper and pencil test provided background information on students’ mathematical thinking development after eleven years of learning mathematics in school. The obtained information from the analyses paints a picture of students’ mathematical thinking development. The Mathematical Thinking Test comprised ten non-routine problems relating to ratio and proportion, algebra, sequence, indices, simultaneous equations, and fundamentals of numbers. Some of the problems used in the study are as follows:

- Find the last digit of $3^{2007}$
- A printer uses 993 digits to number the pages of a book. How many pages are there in the book?
- When the first 97 whole numbers are added, what is the digit in the ones place of this total?

The maximum score for this test was 40, with four marks for each problem. The scoring rubric for the paper and pencil test is shown in Table1.

<table>
<thead>
<tr>
<th>Scoring</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Blank/ No attempt/Illogical attempt</td>
</tr>
<tr>
<td>1</td>
<td>Some elements identified but with inappropriate procedures shown</td>
</tr>
<tr>
<td>2</td>
<td>Identify most elements with appropriate procedures</td>
</tr>
<tr>
<td>3</td>
<td>Identify all elements with completely appropriate procedures but incorrect due to carelessness.</td>
</tr>
<tr>
<td>4</td>
<td>Identify all elements with completely appropriate procedures and correct responses.</td>
</tr>
</tbody>
</table>

A paper and a pencil test was administered to all the participating students with a time allocation of one hour and fifteen minutes. The maximum score for this test was 40, with four marks for each problem. The scoring rubric for the paper and pencil test is shown in Table1.

4.0 Findings

The first section provides the background information of the students involved in the study based on their academic capabilities in the context of mathematics. The subsequent section addressed the three research questions posed for the study.

Table 2 and Table 3 show the mathematics grades obtained by the students in the National examination (SPM) for the subjects of Modern Mathematics and Additional Mathematics.

<table>
<thead>
<tr>
<th>Scoring</th>
<th>Frequency</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>649</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>649</td>
<td>100%</td>
</tr>
</tbody>
</table>

The data shows all students (100%) obtained an A grade in Modern Mathematics, depicting as excellent students involved in the study.

<table>
<thead>
<tr>
<th>Scoring</th>
<th>Frequency</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>77</td>
<td>11.9</td>
</tr>
<tr>
<td>B</td>
<td>263</td>
<td>40.5</td>
</tr>
<tr>
<td>C</td>
<td>309</td>
<td>47.6</td>
</tr>
<tr>
<td>Total</td>
<td>649</td>
<td>100%</td>
</tr>
</tbody>
</table>
Table 3 shows that 52.4% (n=340) of the students are above average based on the A and B grades obtained in the additional mathematics in the SPM examination. To be noted that Modern Mathematics as a subject is taken by all students in secondary school while Additional Mathematics, which is considered higher-level mathematics, is an optional subject taken by students.

Research Question 1. What is the extent of students' achievement in the Mathematical Thinking Test?

Table 4 shows the mean score achieved in the Mathematical Thinking Test among the six hundred and forty-nine students involved in the study.

<table>
<thead>
<tr>
<th>Table 4: Mathematical Thinking Test Scores (Max Score:40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
</tr>
<tr>
<td>649</td>
</tr>
</tbody>
</table>

It depicts that the mean score of the 649 students is a low 10.66 (SD= 7.10), showing a percentage score of 26.7% (10.66/40 x 100). This analysis indicates the high school leavers obtained low-level attainment in the Mathematical Thinking Test.

Research Question 2: Is there a significant impact of Additional Math Grades on students' Mathematical Thinking Test (MTT) scores?

This question analyzed the impact of Additional math test scores as a significant predictor of the Mathematical Thinking test scores. Table 5 shows the regression analysis of the Mathematical Thinking test scores.

<table>
<thead>
<tr>
<th>Table 5 Regression analysis of MTT scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypotheses</td>
</tr>
<tr>
<td>H₁</td>
</tr>
<tr>
<td>R²</td>
</tr>
</tbody>
</table>

The dependent variable (MTT) was regressed on predicting variables of Additional Mathematics grades. The independent variables of AMG significantly impact MTT scores [F(1, 647)=4.583, p = .033] at the 0.05 level. Moreover, the R² = 0.007 depicts that the model explains a meagre 0.7% variance in MTT scores.

4.1 Analysis of Difficulty Faced by Students
The following are samples of students' incorrect responses from the Mathematical Thinking Test (MTT). In light of limitations in terms of space, only a discussion of three questions was undertaken from the MTT based on the highest level of difficulty faced by students. Figure 1 shows a sample of incorrect responses to Question 1. For this question, 47.7% of the students were incorrect, compared to 53.3% with the correct solution. 42.7% (N=277) of them who got it wrong (answer as 4) had utilized mechanical reasoning, as shown in Figure 1. The analyses show faulty algorithm procedures of 14/21 x 6, depicting students' superficial understanding of proportion and ratio. One of the significant issues of concern is students do not check their final answer to ascertain whether it makes sense (or not). The analyses indicate that most of the students obtained four (4) men as the solution as they could not cognize this question as an inverse proportion.

For question 2, students were to find the area of an isosceles triangle without applying the previously learned formula such as Pythagoras or trigonometric functions. The analysis revealed that 73.5% of the students attained an incorrect response due to their lto
nability think beyond the application of formulaic structures. Figure 2a and Figure 2b are samples of the incorrect responses of the students in solving the given problem.

![Figure 2a. Sample of response 1 for Question 2](image)

![Figure 2b. Sample of response 2 for Question 2](image)

The correct solution was obtained through a geometry representation of a square docking a quarter of the formed square (1/4 x 24 x 24 cm²). Samples of the steps as shown in Figure 2a and Figure 2b show over-relying on the formulaic structure of the trigonometry method but were unable to derive the correct solution.

For Question 8, as shown in Figure 3a, 80.8% were mistaken about answering this problem. This question could be related to combination or arithmetic progression studied in high school. None of the students used these two concepts to solve the given problem.

For example, Figure 3a illustrates the difficulty faced by a student, and the student did not attempt to solve this type of question. The majority of students left a blank space due to their inability to cognize the given task.

![Figure 3a. Sample of response 1 for Question 8](image)

The analyses of the solution in Figure 3b are as follows:

![Figure 3b. Sample of response 2 for Question 8](image)

2 students produce 1 handshake
3 students produce 3 handshakes
4 students produce 6 handshakes
5 students produce 10 handshakes
6 students produce 15 handshakes
7 students produce 21 handshakes
8 students produce 28 handshakes
9 students produce 36 handshakes

Algorithmic procedure:

\[ nC2 = \frac{n!}{2!(n-2)!} = 36 \]

\[ n(n-1)/2 = 36 \]

\[ n^2 - n = 72 \]

\[ (n-9)(n+8) = 0 \]

\[ n = 9 \]

None of the students could use the learned algorithmic procedure of combination formulae to solve this problem. Instead, the successful students used heuristics by looking at a pattern in their attempt to solve the problem. As Parmijit et al. (2016) explain, notwithstanding the way that mathematics learning has been progressing over the decades (from elementary to secondary school), students’ lack of cognitive...
strategies, thinking skills, and mathematical aptitude in solving non-routine problems is an area of concern that might inhibit their entry requirement for tertiary level mathematics learning.

5.0 Discussion
The 649 high school leavers are considered high achievers based on their national examination (SPM) grades. However, the Mathematical Thinking Test (MTT) scores obtained in this study depict low-level percentage attainment of 26.7% (10.66/40 x 100). These findings are similar to several previous studies that found that despite scoring A's in year-end maths examinations, most students lack a substantive concept of mathematical thinking (Parmjit et al., 2016; Noor Azina & Halimah, 2009; Mohammadpour et al., 2009). One of the probable causes for this is due to students' lack of exposure to solving non-routine questions in their previous classroom pedagogical practices. Non-routine problems, as its phrase suggests, are those without direct, premeditated answers to a specified problem or task. In such settings, the procedure or method of deriving a solution is not directly attained, subsequently calling for the application of creativity and previously formed knowledge in a novel and unfamiliar condition. Polya (2004) suggested that to develop learners' higher-order thinking, problems in the nature of non-routine should be used. Ridgway, Nickson, and McCusker (2011) stated that "thinking mathematically is about developing habits of mind that are always there when you need them - not in a book you can look up later" (p. 311). This is already designed in an individual's mind and unfolds when resolving problems. Since the students who participated in this study have a low level of mastery in Mathematical Thinking, they lack critical thinking and creative thinking in solving higher-order thinking tasks. These findings were echoed a decade ago (Noor Azina & Halimah, 2009; Mohammadpour, 2009; Parmjit & White, 2006) and are still prevalent today.

Another possible factor contributing to this low attainment in mathematical thinking development among these students might be the mutual pedagogical misconception that "doing mathematics" is similar to being involved in "mathematical thinking." This occurs from the pedantic mathematical education that focuses on the mastery of mathematics by means of rote memorization of formulaic structures, especially from the expression of practice makes perfect. This might be true for mastery skills for arithmetic operations but not for the development of mathematical thinking. This is because in applying the above, students are working towards obtaining the correct answer, and in the process, they tend to side-line considerations such as context, structure, and situations. Furthermore, students lack the opportunity to generate the "richly inter-connected spaces" that Cooper (1998) referred to as forming a vital component in developing mathematical thinking. Students often end up saturated with tonnes of theoretical knowledge that serves no useful purpose. The negative impact flowing from there is perceived when such an approach loses viability as students climb the academic stairways, progressing to higher levels of tertiary education.

Problem-solving lies at the very heart of mathematical learning. The majority of these students also seem to face great difficulty with the solution due to a lack of heuristics application. The lack of these heuristics was evident in this study as most of these students did not have the repertoire of heuristics to aid them in the problem-solving situation. When learners cannot relate the learned math knowledge to solving a problem, heuristics need to be taught formally where it calls for comprehension, understanding of patterns, experimentation, testing of outcomes, and a quest for solutions (Adiguzel & Akpinar, 2004). The heuristics in question are not considered a scientific method of solving a problem or to the regarded as the outcome as elucidated by Polya (2004) but rather to give rise to guidelines of practical reasoning that compel one to think in a manner that would bring about success. These are principles underlying the problem-solving process when it comes to various unfamiliar problems (Posamentier, Smith, & Stepelman, 2006). With scaffolding, they will link and bridge the gap between these unscientific methods with the formal, learned math knowledge.

These findings seem to indicate that with the various reform undertaken by the education ministry, these high school leavers' intellectual capacities are still not on par with the expected level of cognitive demand at the tertiary level. The implication here is that if no remedial actions are taken, the cognitive gap between high school leavers and tertiary levels requirement will widen and these students will be left behind.

6.0 Conclusion & Recommendations
The conclusion from this study can be surmised that these high school leavers had a low-level development of mathematical thinking with an over-reliance on formulas and algorithmic procedures. Their knowledge of mathematical terminology is scarce; they lack understanding of the problem and cannot make representations of unfamiliar problems. The findings also indicate that these high school leavers are not exposed to heuristics as a tool in using strategies in solving the problem and are more focused on solving routine problems based on some formulaic structures and procedures. In a nutshell, the students lacked fundamental math knowledge and could not apply math learned in school to problem-solving situations. However, a limitation of this study is that the outcome was based on a paper a pencil test and thus the cognitive processes involved in students solving and making sense of the problems were not able to be established especially in the context of 'why'. Thus, further research is recommended using qualitative approaches such as interviews to provide information on students thinking process, their understanding and difficulties faced and answers the most important question, why. A new direction of further research is to examine the impact of these low mathematical development through a longitudinal study where it will allow researcher to identify, which specific math thinking skills are lacking among these students, and whether there is a continuing weakness in these skills throughout their years in college.
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References


